Workshop on Geometry of Banach Spaces and its Applications

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Titles and Abstracts

Department of Applied Sciences
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Applications of geometry of Banach spaces in Function theory

T.S.S.R.K. Rao
CARAMS, MAHE

Abstract. This is a teaser talk meant for second year Master students and young researchers . We will gradually move from scalar-valued to vector-valued function theory and demonstrate the power of geometric techniques to solve a basic problem "When are quotients of function spaces, again a function space?" Our ideas and interpretation, take us beyond classical work of Tietz, Dugundji and Edwards.

Numerical index of vector-valued function spaces

Abdullah Bin Abu Baker
IIIT Allahabad

Abstract. The numerical index of a Banach space is a constant relating the norm and the numerical range of operators on the space. This concept was first introduced by G. Lumer in 1968. In this talk, we begin by discussing some properties of numerical range, numerical radius of operators on Hilbert spaces and then Banach spaces. Then we will introduce the concept of Numerical index of a Banach space, and discuss some recent results.

Strong proximinality in L_1-predual spaces

C. R. Jayanarayanan
IIT Palakkad

Abstract. In this talk, we discuss the strong proximinality of subspaces and closed unit ball of subspaces of L_1-predual spaces. We also discuss the continuity properties of metric projection in L_1-predual spaces.

On Uniform Mazur Intersection Property

Pradipta Bandyopadhyay
ISI Kolkata

Abstract. The Mazur Intersection Property (MIP) — every closed bounded convex set is the intersection of closed balls containing it — is an extremely well studied property in Banach space theory. A complete characterisation was obtained by Giles, Gregory & Sims (1978), the most well-known criterion stating that the w^*-denting points of $B(X^*)$ are norm dense in $S(X^*)$. Chen and

Lin (1995), introduced the notion of w^*-semidenting points and showed that a Banach space X has the MIP if and only if every f in $S(X^*)$ is a w^*-semidenting point of $B(X^*)$.

A much less studied uniform version of the MIP (UMIP or UI) was introduced by Whitfield and Zizler (1987). Characterisations similar to Giles, Gregory & Sims were also obtained, but an analogue of the w^*-denting point criterion was missing, which perhaps is a reason for its being less pursued.

In this talk, we show that a Banach space X has the UMIP if and only if every f in $S(X^*)$ is a uniformly w^*-semidenting point of $B(X^*)$, thus filling a long felt gap. Proof of the main result will be presented.

This is a joint work with Jadav Ganesh & Deepak Gothwal.

Role of Birkhoff-James orthogonality in the geometry of Banach spaces

Debamalya Sain
IISc Bangalore

Abstract. I plan to discuss the importance of Birkhoff-James orthogonality in studying the geometry of Banach spaces, with illustrative examples and possible scopes of application. Particular attention will be paid to the notion of smoothness. If time permits, I would also like to touch upon the utility of Birkhoff-James orthogonality techniques in norm optimization problems.

Hahn-Banach smoothness and its various strengthening and weakening

Soumitra Daptari
IIT Hyderabad

Abstract. The Hahn-Banach extension theorem is very well known to us and it guarantees a norm-preserving extension of a bounded linear functional on a subspace Y of a normed linear space X. The subspace Y of X is called Hahn-Banach smooth (or said to have property-(U)) if every $g \in Y^*$ has a unique norm preserving extension to X. In this talk, we look at the property-(U), some ways to strengthen the property-(U), such as property-(SU) and property-(HB), and some ways to weaken it, such as property-(wU) and property-$(k-U)$, where k is a natural number. At the end of the talk, we will discuss a few examples in classical Banach spaces with these properties.

Of Quotient Maps and their Liftings

Amin Sofi
Jammu and Kashmir Institute of Mathematical Sciences

Abstract. One of the elementary and fundamental facts taught in a first course on functional analysis is the Open Mapping Theorem(OMT) which states that a surjective continuous linear map $f : X \to Y$ acting between Banach spaces X and Y is an open map. A direct consequence of this fact is the inverse mapping theorem, asserting that a bijective continuous linear map acting between Banach spaces has a continuous inverse $g : Y \to X$ $(gf = I_X,\ fg = I_Y)$.

This raises the following very natural question that has spawned a great deal of research in recent years involving the so-called non-linear geometry of Banach spaces:

Q.1: Under the conditions of the open mapping theorem, does it follow that $\exists\ g : Y \to X$ with $fg = I_Y$ having better 'regularity' properties?

The case when g can be chosen to be linear and continuous such that $fg = I_Y$ holds exactly when Y is complemented in X. However, thanks to Michael's selection theorem, requiring g to be continuous comes for free - that's always the case!

The fact that there are situations where it is not possible to choose g enjoying better continuity properties, say Lipschitz or uniformly continuous turns out to be important in the "Nonlinear geometry of Banach spaces.

In this talk, we shall address the implications of the existence or otherwise of nice liftings of a quotient map. While it's known that for uniformly convex Banach spaces X, g can always be chosen to be Lipschitz on the unit ball of X/M which, in the case of Hilbert space X turns out to be linear (and continuous) on all of X/M, it's not clear how to specify the exact conditions on a given closed subspace M of X so that the quotient map $f : X \to X/M$ admits a Lipschitz right inverse g. However, it's possible to show that if indeed g can be chosen to be continuous for *each* M, then X is (isomorphically) a Hilbert space.

Nyström Method for the Numerical Solution of Integral Equations and its Application for the Wave Scattering Problems

Ambuj Pandey
IISER Bhopal

Abstract. Owing to the advent of modern computers and fast summation techniques, the integral equation-based approach for the numerical solution of the partial differential equation became very popular in the last few decades. This talk will discuss the Nystrom/Quadrature method and its convergence analysis for the numerical solution of the Fredholm integral equation of the second kind that models many significant real problems arising from science and engineering. We will also present some of the popular fast quadrature

schemes for the numerical solution of integral equations that govern the wave propagation problems as an application of the method.

Some geometric properties of relative Chebyshev centres in Banach spaces

Tanmoy Paul
IIT Hyderabad

Abstract. We will explore two geometric features of Banach spaces in this talk: Property-(P1) and Property-(R1). We will begin with some motivation to learn and then move on to some recent good developments for these properties.

Some characterizations of uniformly convex spaces

Vamsinadh Thota
NIT Tiruchirappalli

Abstract. In this talk, we will discuss various characterizations of uniformly convex spaces in terms of subsets of the closed unit ball and uniform strong proximinality of subspaces.

Greedy Approximations

Divya Khurana
IISc Bangalore

Abstract. Greedy Approximation Algorithms were introduced by Konyagin and Temlyakov in order to study a special class of bases in Banach spaces. These bases are known as greedy bases. A greedy basis can be characterized by the unconditional and democratic properties of the basis. Some of the classical Banach spaces fail to have an unconditional basis. This fact motivated many researchers to study weaker versions of greedy algorithms. In this talk, we will start with some nice properties of Hilbert spaces. Motivated by these properties we will study greedy algorithms and some of their weaker versions.

Algebraic reflexivity of isometries on Banach spaces

Abdullah Bin Abu Baker
IIIT Allahabad

Abstract. Whether local actions of a class of transformations on a given operator algebra determine the class completely is a very basic problem which is investigated by many researchers. We will define the concepts of local maps and algebraic reflexivity and find out the structure of local maps in some special cases.

Dentability, slices and geometry of Banach spaces

Sudeshna Basu
Ramakrishna Mission Vivekananda Educational and Research Institute

Abstract. In this talk we look at how analytical and geometrical properties of Banach spaces are closely linked with each other. We introduce dentability, slices and explore their role in determining the underlying geometry of a Banach space.

On commutative and noncommutative Gurariĭ spaces

Aryaman Sensarma
ISI Bangalore

Abstract. In this talk, we discuss about commutative and noncommutative Gurariĭ spaces. We present that, a (non-separable) Banach space is a Gurariĭ space if and only if every separable a.i.-ideal in X is isometric to the separable Gurariĭ space $\mathbb{G}$. We also obtain a similar characterization of L_1-predual spaces in terms of ideals. Along the way, we show that the family of ideals/a.i.-ideals in a Banach space is closed under increasing limits. We show that if X is almost isometric to a Gurariĭ space, then it is also a Gurariĭ space, thus answering a question of Prof. T. S. S. R. K. Rao. We prove an analogue of this result for the noncommutative Gurariĭ space, as well as for $\mathcal{E}$-Gurariĭ property, and $\mathcal{E}$-injective property. This talk is based on two joint works, first one is with Prof. Pradipta Bandyopadhyay & Prof. Sudipta Dutta and second one is with Prof. Pradipta Bandyopahyay.

Ball proximinality of M-embedded spaces

Sreejith Siju
IIT Palakkad

Abstract. This talk primarily focuses on the ball proximinality of M-embedded spaces. We will see how a method developed to tackle the ball proximinality of space of compact operators can be used to solve the ball proximinality of M-embedded spaces. We will also consider a stronger version of ball proximinality and examine whether the space of compact operators on Hilbert space satisfy this stronger notion.

Projections in convex hull of isometries on the space of continuously differentiable functions

Rahul Maurya
IIIT Allahabad

Abstract. A projection P on a Banach space X is called a generalized bi-circular projection if there exists a unit modulus complex number λ different from 1 such that $P + \lambda(I - P)$ is an isometry on X. In this talk, we shall discuss some properties of generalized bi-circular projections and their relationship with Hermitian projections. We will also characterize projections can be written as convex combination of isometries on the spaces of continuously differentiable functions.